Model Examination paper-I

1(i)Consider a random experiment with three possible outcomes with probabilities p_1, p_2, p_3 respectively. Suppose the above experiment is repeated m times in an independent fashion and $X_i,\ i = 1, 2, 3$ denote the number of times ith outcome occurs.

(a) What is the pmf of $X_1 + X_2$?

(b) Find $P\Big[X_2 = y | X_1 + X_2 = z\Big],\ y = 0, 1, \ldots, z,\ z = 0, 1, \ldots, m$. [6]

1(ii) Let a point be chosen uniformly from the interval $[0,\ a],\ a > 1$. Let X denote the distance of the point chosen from 0 and $Y = \min\{X, \frac{a}{2}\}$. Find the distribution of Y. [4]

2(i) Define a σ-field. Give examples of two σ-fields are which are not independent. [4]

2(ii) Let $A, B, C \in \mathcal{F}$ be independent events in a probability space $(\Omega,\ \mathcal{F},\ P)$. Prove or disprove the following. $\sigma(A)$ is independent of $\sigma(\{B, C\})$. [6]

3(i) Let X be a standard normal random variable. Find

$$E\Big[\int_0^\infty \frac{\sin(tX)}{t} dt\Big]. \qquad [5]$$

3(ii) Let ϕ_1 and ϕ_2 be characteristic functions. Define ϕ_3 and ϕ_4 as follows.

$$\phi_3 = \phi_1\phi_2,\ \phi_4 = \phi_1 + \phi_2.$$

Show that $\phi 3$ is a characteristic function and ϕ_4 is not a characteristic function. [5]

4. Let $X_n \to X$ in probability.

(i) Show that $|X_n| \to |X|$ in probability. [5]

(ii) Is it true that $\max\{X_n, 0\} \to \max\{X, 0\}$ converges in probability? Justify your answer. [5]

5(i) State strong law of large numbers. [3]

5(ii) Let $X_1, X_2, \ldots$ be independent random variables such that

$$P\{X_n = 3^n\} = P\{X_n = -3^n\} = \frac{1}{2},\ n \neq 1\,.$$

Show that $\frac{X_1+X_2+\cdots+X_n}{n}$ does not converge to 0. Does this contradicts strong law of large number? [7]